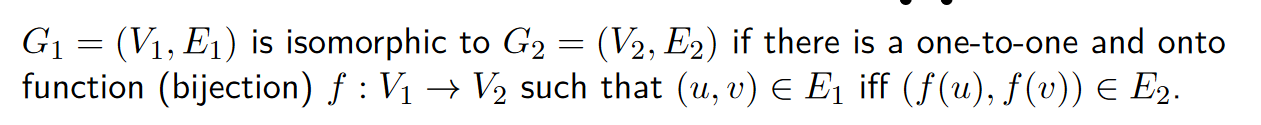
**Isomorphism proof definition as shown in lectures:**



**Written definition of isomorphism:**

1. Equal number of vertices.

2. Equal number of edges.

3. Same degree sequence

4. Same number of circuit of particular length

**Question (a).** Prove that if two graphs A and B do not have the same number of nodes, they cannot be isomorphic. (3 points)

**Informal Proof:** Condition 1 is not satisfied, not all vertices can be matched by a bijective function.

**Formal Proof:**

Let Graph A = {V, E} and B = {V', E'}

Let graph A have n nodes and b have less than n nodes

Let F be a bijective function from V' to V

|  |  |
| --- | --- |
| (For All a,b in V) V = F(V') | Definition of isomorphism of vertices |
| V1 = F(V'c1) ^ V2 = F(V'c2) ^ ... ^ Vn = F(V'cn) | Expansion of (For All a,b in V) where ‘cn’ are separate constants |
| Vn = F(V’cn) | Assume all instances of V = F(V’cn) true |
| False | V’cn does not exist, because b has < n nodes |
| A and B are not isomorphic | A bijective function cannot satisfy the equation, and F must be a bijective function. |

When applying this proof, assign graph A to be the graph with more nodes.

**Question (b)** Prove that if two graphs A and B have the same number of nodes and are completely connected, they must be isomorphic. (3 points)

**Informal Proof:** If both graphs have the same number of nodes and are completely connected, then by definition they satisfy all four conditions. This is because there is only one type of completely connected graph for any given number of nodes- the one where you connect all of the nodes.

**Formal Proof:**

Let graphs A and B have ‘n’ nodes and be completely connected.

Therefore, all nodes are of degree n-1, by definition of completeness.

Therefore, there are n choose 2 edges, by definition of completeness.

Therefore, (1) they have the same number of vertices.

Therefore, (2) they have the same number of edges.

Therefore, (3) both graphs have identical degree sequences, as all nodes have the same degree.

Therefore, (4) both graphs have the same number of circuits of a particular length, as they contain identical circuits.

All conditions are satisfied, the graphs are isomorphic.

**Question (c)** Prove that if two graphs A and B are isomorphic they do not have to be completely connected. (3 points)

Isomorphism is defined separately from connectively, a graph can be isomorphic and completely connected, as well as isomorphic and not completely connected.

**Proof by example:** The following trivial graphs are not completely connected, and are isomorphic graphs. Therefore, if two graphs A and B are isomorphic they do not need to be completely connected.

